Entry and Exchanges of Cost Information

Olivier Armentier* and Oliver Richard†

October 2002

Abstract

The literature on exchanges of information has ignored firms’ entry decisions. Yet, the Federal Trade Commission recently expressed concerns that exchanges of information in business-to-business electronic platforms would adversely impact entry and, thus, consumers. When entry decisions are endogenized in a competitive Cournot model with cost uncertainty, we find results that contrast sharply with current thinking on the welfare consequences of information sharing.

* Dept of Economics, SUNY Stony Brook NY, 11794. E-mail: olivier.armentier@sunysb.edu.
† Simon School of Business, University of Rochester, Rochester, NY 14627; E-mail: richard@ssb.rochester.edu.
‡ We are very grateful to Michael Crew (the editor), two anonymous referees, Greg Shaffer, Jim Brickley, Pamela Bedore, and Scott Stoness for detailed, insightful comments on earlier drafts. We also thank seminar participants at SUNY-Stony Brook and the University of Rochester for suggestions.
1. Introduction

In 1994, the European Commission rejected an agreement to exchange information proposed by a tractor trade association in the U.K. on the ground that it raised barriers to entry and, thus, negatively impacted consumer welfare.\(^1\) The Federal Trade Commission recently expressed the same concerns regarding exchanges of information in business-to-business (hereafter B2B) electronic marketplaces.\(^2\) In their literature review for the European Commission, Kuhn and Vives (1995) report that under Cournot competition a complete sharing of cost information raises firms’ profits, but is harmful to consumers in oligopoly structures with fewer than 10 firms.\(^3\) This literature, however, does not fully address the policy-makers’ concerns since it ignores firms’ entry decisions.

When entry decisions are endogenized, we find results that contrast sharply with the existing literature. For instance, firms may not always have an incentive to exchange cost information. This result holds when an agreement to pool information encourages additional entry on the market which, in turn, lowers expected profits per firm. We show that in these cases, social welfare may nevertheless be higher if the firms do exchange information. In other words, we identify some situations where policy-makers should entice

---


\(^2\) B2B electronic marketplaces use the Internet to connect businesses to each other in order to buy and sell goods, but also exchange information. The Federal Trade Commission in its October 2000 report entitled “Entering the 21st Century: Competition Policy in the World of B2B Electronic Marketplaces” (available on the Commission’s web site) predicts that purchases from B2B will grow to $1.4 trillion by 2002.

firms to exchange information. We then highlight that it may be possible to encourage this exchange without any monetary transfers. Finally, we find that, even if the number of entrants is the same whether or not firms exchange information, then consumers may benefit in market structures with as few as two firms. These results are interesting from a regulatory perspective, since policy-makers typically consider consumer welfare to be the deciding factor in antitrust cases.

The paper is structured as follows. We describe the standard oligopoly model in Section 2 and generalize it slightly in Section 3. We then introduce the model with entry in Section 4 and analyze its implications. Section 5 concludes.

2. The Standard Oligopoly Model

Exchanges of cost information have usually been analyzed under a standard oligopoly model (hereafter SOM) originally proposed by Fried (1984), Shapiro (1986) and then extended by Sakai and Yamato (1989) and Raith (1996). We now summarize the main features of this model.

There are $N$ firms ($i = 1, ..., N$) competing on a single market, each producing a differentiated product. Let $x_i$ be the output of the $i^{th}$ firm, and $p_i$ its unit price. On the demand side, there is a representative consumer with the following quadratic utility function:

$$U(x_0, x_1, ..., x_N) = x_0 + a \sum_{i=1}^{N} x_i - \frac{1}{2} \left( \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} \sum_{j \neq i} x_j x_i \right)$$  (2.1)
where $x_0$ is the numeraire good, $a > 0$ and $0 \leq b \leq 1$. The representative consumer maximizes his surplus, $U(x_0, x_1, \ldots, x_N) - x_0 - \sum_{i=1}^{N} p_i x_i$, which leads to the following system of inverse demand functions:

$$p_i = a - x_i - b \sum_{j \neq i}^{N} x_j \quad \forall i = 1, \ldots, N.$$  \hspace{1cm} (2.2)

These demand functions are assumed to be common knowledge.

Marginal costs of production, $c_i$ ($i = 1, \ldots, N$), are typically assumed to be jointly normally distributed with $E(c_i) = \mu > 0$, $Var(c_i) = \sigma^2$, $Cov(c_i, c_j) = \rho \sigma^2$ $\forall i \neq j$ and $-1/(N-1) \leq \rho \leq 1$.\(^4\) Marginal costs are assumed to be private information. In other words, each firm knows its own marginal cost but does not observe its rivals'.\(^5\)

The game consists of three stages:

- **Stage 1**: Firms jointly decide whether or not to enter a binding agreement to exchange cost information. Under this agreement, firms truthfully reveal to each other their cost vector $c_i$ at the beginning of stage 2. It is assumed that firms can transfer and verify each other’s reports at no cost. Renegotiations after stage 1 are not allowed.\(^6\)

\(^4\)See, for instance, Fried (1984), Gal-Or (1986), Raith (1996), Shapiro (1986), Sakai and Yamato (1989), and Vives (1999). Note that the normality assumption is chosen in the literature because it yields affine conditional expectations (e.g. $E(c_j | c_i) = \rho(c_i - \mu) + \mu$ $\forall j \neq i$) and analytical tractability.

\(^5\)In the context of the SOM, it is technically equivalent to have the constant marginal cost of production or a firm-specific demand intercept ($\alpha_i$) be private values to the firm. Note that some models assume that firms only observe a noisy signal $\tilde{c}_i = c_i + \varepsilon_i$ where $\varepsilon_i$ is an error term (see Raith (1996)).

\(^6\)Okuno-Fujiiwara et al. (1990) and Ziv (1993) show that firms may want to renegotiate their agreement to exchange information after learning their type. We assume that a time constraint associated with the approval of a cost sharing agreement by a regulatory commission prevents any renegotiation after stage 1.
• **Stage 2:** If all firms agreed to share information in stage 1, then each firm observes the entire vector of marginal costs \((c_1, ..., c_N)\).\(^7\) Otherwise, firm \(i\) only observes its own marginal cost \(c_i\). Given the information structure on costs, each firm then chooses its output level so as to maximize its (expected) profits.

The model is solved using backward induction. In stage 3, the symmetric Nash equilibrium quantity produced by a firm under a cost sharing agreement (i.e., competition under complete information) is

\[
x_{i,s}^* (c_i) = \frac{a (2 - b) - c_i (2 + (N - 2) b) + b \sum_{j \neq i} c_j}{(2 + (N - 1) b) (2 - b)}.
\]

(2.3)

On the other hand, when firms decide not to enter any agreement, they compete under incomplete information on marginal costs and produce at the equilibrium

\[
x_{i,ns}^* (c_i) = \frac{a - \mu}{2 + b(N - 1)} - \frac{(c_i - \mu)}{2 + b(N - 1) \rho}.
\]

(2.4)

Raith (1996) shows that firms’ expected profits are larger under complete information. Therefore, the subgame perfect Nash equilibrium strategy is to agree to share cost information in stage 1 and to choose quantity in stage 3 according to equation (2.3).

The effect of cost sharing on consumers is not as straightforward, however. Sakai and

---

\(^7\)To the best of our knowledge, only Shapiro (1986) analyzes information sharing among subsets of firms. Given the hypothesis that products are perfect substitutes, he shows that all firms exchange cost information in equilibrium. Note, as well, that some authors assume that firms only observe a noisy version of their competitors’ marginal costs: \(\tilde{c}_j = c_j + \varepsilon_i \forall j \neq i\) where \(\varepsilon_i\) is an error term (see Gal-Or (1986)).
Yamato (1989) show that there exist some pairs \((b, \rho) \in [0, 1]\) such that the expected consumer surplus is larger under information sharing, but this is possible only if costs are correlated and \(N \geq 10\). In other words, information sharing is harmful to consumers in oligopoly structures with fewer than 10 firms. Nevertheless, Sakai and Yamato (1989) show that information sharing on costs always improves social welfare in the SOM.

The key to understand intuitively the results for the firms in the SOM is to realize that, whether or not information is shared, a firm’s expected profit equals:

\[
\frac{1}{2} \left[ \text{Var} \left[ x_i^* \right] + E \left[ x_i^* \right]^2 \right]
\]

(2.5)

where \(x_i^*\) is the equilibrium quantity of firm \(i\). Information sharing therefore benefits firms, as it does not affect expected equilibrium quantities (i.e. \(E \left[ x_{i,s}^* \right] = E \left[ x_{i,ns}^* \right]\) given (2.3) and (2.4)), while it increases the variance of each firm’s equilibrium output (i.e. \(\text{Var} \left[ x_{i,s}^* \right] \geq \text{Var} \left[ x_{i,ns}^* \right]\)). To see this, consider for instance a Cournot duopoly in which firms know their own cost, but not their rival’s. If the firms do not exchange information, then each firm produces, after observing its own cost, a single equilibrium quantity, irrespectively of its rival’s actual cost. However, if a quantity-setting firm learns that the cost of its rival is higher (lower) than expected, then it will produce more (less) in equilibrium. These strategic adjustments to the true state of the world under information sharing increase the variability in individual equilibrium outputs (hereafter referred to

---

8Whether or not information is shared, firm \(i\)'s expected profit equals \(\frac{1}{2}E \left[ x_i^{*2} \right] = \frac{1}{2} \text{Var} \left( x_i^{*2} \right) + E \left[ x_i^* \right]^2\), where \(x_i^*\) is the equilibrium quantity. The expression \(\frac{1}{2}E \left[ x_i^{*2} \right]\) obtains by substituting the firm’s equilibrium quantity derived from the first order conditions into the profit function.
as own-variation effect), which is beneficial to firms. The gains from information sharing are larger when the firms produce similar goods, but know little about each other’s costs. In other words, exchanges of information are more valuable to firms when cost signals are uncorrelated (i.e. $\rho$ is close to 0), and when products are close substitutes (i.e. $b$ is close to 1), in which case a firm’s profitability is more affected by its competitors’ choices.

With regards to the consumer, we note that the expected consumer surplus equals:

$$
\frac{1}{2} \sum_{i=1}^{N} \left( \text{Var} \left[ x_i^* \right] + E \left[ x_i^* \right]^2 \right) + \frac{b}{2} \sum_{i=1}^{N} \sum_{j \neq i}^{N} \left( \text{cov} \left( x_i^*, x_j^* \right) + E \left[ x_i^* \right] E \left[ x_j^* \right] \right). 
$$

As previously explained, exchanges of information do not affect the expected quantities, but they increase the variance of each firm’s equilibrium output. Equation (2.6) indicates that this own-variation effect is beneficial to the consumer. However, information sharing also lowers the covariances between equilibrium outputs, and this negatively impacts consumers since the expected consumer surplus in (2.6) is increasing in the covariances. Indeed, firms sharing information adjust their strategy to the true state of the world, so that in equilibrium, high cost firms produce less, and low cost firms produce more. These strategic adjustments are such that the covariance between pairs of equilibrium outputs decreases (hereafter referred to as cross-variation effect). Consequently, the

---

9The expected consumer surplus equals $E \left[ U \left( x_0, x_1, \ldots, x_N \right) - x_0 - \sum_{i=1}^{N} p_i x_i \right]$. After substituting (2.1) and (2.2) into the previous expression, we get $\frac{1}{2}E \left[ \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} \sum_{j \neq i}^{N} x_j x_i \right]$, which in equilibrium leads to (2.6).

10Note that, whether or not information is exchanged, the covariance of any two substitute goods in the SOM is negative since firms’ reaction functions are negatively sloped.
expected consumer surplus may be lower under information sharing when the cross-variation effect is pronounced; that is, in markets with few firms and high product differentiation (see Sakai and Yamato (1989)).

Finally, once firms’ profits and consumer surplus are jointly considered, information sharing always increases social welfare (see Sakai and Yamato (1989)). In other words, the benefits of the own-variation effect to firms and consumers may be shown to outweigh the negative impact of the cross-variation effect on consumers.

3. Comments and Generalization

The SOM relies upon two implicit assumptions:

- **Assumption 1**: The number of firms on the market is fixed to $N$.

- **Assumption 2**: The number of firms on the market and the parameters of the model are such that the probability that firms produce non-positive equilibrium quantities is negligible.$^{11}$

Assumption 1 implies that entry decisions are exogenous and, in particular, independent of any agreement to exchange information. In other words, the number of firms on the market is the same whether or not firms decide to exchange information. A consequence of Assumption 2 is that the model may be solved as if all firms are going to produce strictly positive quantities in equilibrium.

$^{11}$One can trivially verify that there exist some parameter values such that the optimal strategies in (2.3) and (2.4) are negative.
The objective of the present paper is to relax assumption 1 and to examine the effects of exchanges of cost information on entry decisions. However, in models with entry, the probability of firms producing negative equilibrium quantities cannot always be assumed to be negligible, since the number of firms varies endogenously. To see this, note that expected profits in both information structures are equal to $E[(x_{i,d}^*(c_i))^2]$, $d \in \{s, ns\}$, and therefore they cannot be negative. Thus, the number of entrants may become unbounded when costs of entry become arbitrarily small. Consequently, the probability of negative equilibrium quantities cannot be ignored as it increases with the number of firms. Hence, we first generalize the SOM by adding a non-negativity constraint on quantities choices.\footnote{Since the distribution of costs is unbounded, marginal costs and prices may also be negative. The probability of negative costs is negligible for an appropriate selection of parameters ($\mu, \sigma$) and the results in sections 3 and 4 generalize to any non-negative continuous distribution on costs. The resolution of the model under an additional non-negativity constraint on prices is truly non-trivial. In the subsequent numerical simulations, the parameters of the model are such that costs and prices are always positive.}

3.1. A Generalized Standard Oligopoly Model

In the private (or incomplete) information structure, firm $i$ now chooses a weakly positive output level so as to maximize its expected profits:

$$x_{i,ns}^*(c_i) = \arg\max_{x_i} E[\pi_i,ns|x_i|c_i] = E[(p_i - c_i)x_i|c_i] \quad \text{subject to} \quad x_i \geq 0.$$
The corresponding symmetric Nash equilibrium quantities are given by:

\[ x_{i,n}^*(c_i) = \frac{1}{2} \left( a - b \left( N - 1 \right) E[x^*|c_i] - c_i \right) I_{\{c_i \leq \bar{c}\}} \quad \forall i = 1, \ldots, N \quad (3.1) \]

where \( \bar{c} = a - b \left( N - 1 \right) E[x^*|\bar{c}] \) . \( I_{\{c_i \leq \bar{c}\}} \) is the indicator function defined as \( I_{\{c_i \leq \bar{c}\}} = 1 \) when \( c_i \leq \bar{c} \) and \( I_{\{c_i \leq \bar{c}\}} = 0 \) otherwise; \( \bar{c} \) is a threshold cost value and \( E[x^*|c_i] \) is the conditional expected equilibrium quantity of any competitor of player \( i \). Note that firms only produce in equilibrium if their marginal cost is below the threshold cost \( \bar{c} \).

When firms exchange information, the optimization problem is:

\[ x_{i,s}^*(c_i) = \arg \max_{x_i} \pi_{i,s} = (p_i - c_i) x_i \quad \text{subject to} \quad x_i \geq 0 \]

which leads to the following equilibrium quantities:

\[ x_{i,s}^*(c_i) = \left[ \frac{2 + b \left( \sum_{j \neq i}^{N} I_{\{c_j \leq \bar{c}_j\}} - 1 \right) }{(2 - b) \left( 2 + b \left( \sum_{j=1}^{N} I_{\{c_j \leq \bar{c}_j\}} - 1 \right) \right) } \left( \bar{c}_i - c_i \right) \right] I_{\{c_i < \bar{c}_i\}} \quad (3.3) \]

where \( \bar{c}_i = \frac{(2 - b) a + b \sum_{j \neq i}^{N} c_j I_{\{c_j \leq \bar{c}_j\}}}{2 + b \left( \sum_{j \neq i}^{N} I_{\{c_j \leq \bar{c}_j\}} - 1 \right) } \quad \forall i = 1, \ldots, N \quad . \quad (3.4) \]

The optimal solutions in both information structures are non-linear and they rely upon
a system of implicit equations. We propose in the Appendix a general algorithm to
determine numerically the constrained Nash equilibrium quantities. The object of this
algorithm is to solve numerically the implicit equations defining the different \( \tau_i \) and to
approximate the function \( E [x^*|c_i] \) using simulations.

We find that the addition of a non-negativity constraint preserves firms’ incentive to
share information. The intuition is basically the same as in the SOM: under information
sharing, firms adjust their equilibrium outputs to the true state of the world, so that
low cost firms produce more and high cost firms produce less. As explained in Section
2.1, these output adjustments increase the variability in each firm’s equilibrium output
(i.e. own-variation effect) and, thus, firms’ expected profits. However, in a model with
constraints, information exchanges may not lead all firms to modify their strategy. For
instance, a high-cost firm may remain inactive whether or not it learns its rivals’ costs.\(^{13}\)
As a consequence, the own variation effect is reduced, and gains in expected profits from
information sharing are lower than in the SOM. Finally, as in the SOM, the gains from
information sharing are larger when information is most valuable to firms; that is, when
the correlation in costs is low and products are close substitutes.

We also find that when consumers benefit from exchanges of information in the SOM,
ye also benefit in the generalized SOM. More significantly, we identify a large number
of instances where consumers benefit from information sharing in market structures with
less than ten firms and as few as two firms, in sharp contrast with findings for the SOM

\(^{13}\)A firm is said to be active when it produces a strictly positive quantity in equilibrium. Likewise, a
market is said to be active when at least one firm is active.
(where we must have $N \geq 10$ for consumers to benefit).

To illustrate, consider the following examples where costs are jointly normally distributed, $a = 10$, $\mu = 10$, $\sigma^2 = 2$. Specific numerical results for $(N = 2, b = 0.5, \rho = 0.95)$ and $(N = 10, b = 0.5, \rho = 0.7)$ are summarized in Tables 1 and 2. These results indicate that firms increase their expected profits by exchanging information, albeit the gains are slightly smaller (in absolute and relative terms) in our generalized model.\textsuperscript{14}

Graphs 1 and 2 illustrate that consumers benefit from information sharing for a larger set of parameter values $(N, b, \rho)$ than implied by the SOM. Intuitively, as explained in Section 2.1, information sharing in the SOM increases both the variability in firms’ individual equilibrium outputs and the disparity in equilibrium outputs across firms. The first effect (i.e. own-variation) benefits consumers, but the second (i.e. cross-variation) hurts them. These two effects are, once again, present in the generalized SOM. However, there is now a third effect that works to the consumers’ advantage. Indeed, when firms share information in the generalized SOM, they are able to better decide who should enter a market. Consequently, a market is active more frequently under information sharing, which is beneficial to consumers. This effect is particularly relevant when products are close substitutes (i.e. $b$ is close to 1) and information is most valuable to firms. As graphs 1 and 2 illustrate, the combination of the three effects (i.e. increase in market activity and own-variation on one hand, and cross-variation on the

\textsuperscript{14}We define a variable $ENTRY$ such that $ENTRY_{i,j} = 1$ when firm $i$ enters the market at simulation $j$ and $ENTRY_{i,j} = -1$ otherwise. In Table 2, although the probability that a given firm produces a positive quantity may be lower under complete information than under incomplete information, entry decisions are less correlated.
other) is such that consumers may benefit from information exchanges in the generalized SOM in market structures with as few as two firms, when \((\rho, b)\) are close to \((1, 1)\). This result contrasts with findings in the SOM.

4. The Model with Entry

We now extend the traditional model to examine the effects of exchanges of information on entry decisions. The structure of the model is as follows:

- **Stage 1**: The \(N\) firms simultaneously decide whether or not to enter into a binding agreement to exchange cost information.

- **Stage 2**: Each firm chooses whether or not to enter the market. If firm \(i\) enters the market it incurs a fixed costs \(F_i\). Fixed costs are assumed to be common knowledge and sunk upon entry.

- **Stage 3**: If the entrants agreed to share cost information in stage 1, then they observe the entire vector of marginal costs. Otherwise, each entrant observes only its own marginal cost. Then, given the information structure on costs, the firms simultaneously choose their output level, subject to the constraint that they produce a non-negative quantity. In other words, should a firm have a high marginal cost given the actual number of entrants, it may choose not to supply the market by selecting an output of zero.
The fixed costs may include foreseeable expenses associated with investments in capacity, R&D, contract or licensing fees, and development of sales, supply, or distribution channels. These fixed costs may be incurred by new firms entering a market or by incumbents to renew or improve their access to the market. Exchanges of information may therefore take place between incumbents and/or new firms. Marginal costs, on the other hand, are inherently affected by factors such as input prices, workers’ productivity or unforeseen idiosyncratic events such as delays. In this context, a firm precisely observes its marginal cost only at the time of production.

The structure of this model may characterize R&D intensive industries (e.g. semiconductors), industries with sunk capacity investments (e.g. steel), or industries where extensive marketing or distribution channels are necessary (e.g. airlines (see Kuhn and Vives (1995), or automakers (see Doyle and Snyder (1999))). In these industries, trade associations and joint ventures have historically been the mediums for exchanges of information. In recent years, however, B2B e-commerce internet platforms have both facilitated and widely expanded exchanges of information.

The timing of our model is consistent with firms’ practices and optimal behavior in the current regulatory environment. Indeed, as indicated by Kuhn and Vives’ 1995 report to the EEC, and the FTC’s 2001 review of B2B internet marketplaces, policymakers have long shown an interest in reviewing agreements to exchange information, as they may reduce consumer welfare. Therefore, firms exchanging information face the likelihood of a potentially lengthy, costly and uncertain review process. In this context, firms whose profitability is conditional on an exchange of information should
find it optimal to announce their intentions to share information prior to incurring the sunk expenses related to entry. Recent evidence on B2B marketplaces supports this view. For instance, when the major automakers considered exchanging information on prices and availability of supply parts through a B2B internet marketplace (the Covisint project), they first publicly announced their intentions, then awaited the Federal Trade Commission’s guarded approval before proceeding with foreseeable production costs.\footnote{Concise overviews of the Covisint decision may be found in “B2B Exchanges Get Yellow Antitrust Light” from the law firm Katten Muchin Zavis, available at www.kmnz.com, and on Professor Jack M. Wilson’s web site at www.jackwilson.com/ebusiness/.}

## 4.1. Exchanges of Information: Firms’ Incentives

We now show that, in contrast to the findings for the SOM, firms may not find it profitable to exchange cost information in a model with entry.

Rank, without loss of generality, firms according to their fixed cost $F_1 \leq F_2 \leq \ldots \leq F_N$. In stage 2, firms have not yet observed their marginal cost. Hence, firms are symmetric and expected (variable) profits in stage 3, which are conditional upon the number of entrants and the information structure, are identical across firms. Let $E[\pi_d|n]$ denote a firm’s conditional expected profits in stage 3 given information structure $d$ ($d \in \{s, ns\}$) and $n$ entrants. Then let $n_d^* = \sup_{1 \leq n \leq N} (n : E[\pi_d|n] - F_n \geq 0)$ be the number of firms that have non-negative net expected profits in stage 2 under information structure $d$.

As conditional expected profits in stage 3 strictly decline with the number $n$ of entrants, $n_s^*$ and $n_{ns}^*$ are uniquely defined.\footnote{See Berry (1992) for a proof of this result.} However, the identity of the $n_q^*$ entering
firms need not be unique, as when firms $n_q^*$ and $n_q^* + 1$ are such that one or the other (but not both) have positive expected profits in the presence of firms 1 to $(n_q^* - 1)$. We assume that the identity of the firms in $n_s^*$ and $n_{ns}^*$ is unique and concentrate on the socially efficient Nash equilibrium in pure strategies. Under this solution concept, the firms that enter in the equilibrium are the ones with the smallest fixed costs.

We saw in Section 3 that, for a given number of firms, expected profits are higher under information sharing. It follows, from the definition of $n_q^*$, that the number of entrants in stage 2 is at least as large under an agreement to exchange information as under no agreement (i.e., $n_s^* \geq n_{ns}^*$). Hence, we need to consider three cases:

- (4.1.1) If $n_s^* = n_{ns}^*$, then firms exchange cost information.

If the number of entrants is the same under both information structures, then the model is similar to the generalized SOM developed in Section 3. In that section we have seen that for a given number of firms, expected profits are higher under information sharing. Therefore, in the equilibrium, firms 1 through $n_s^*$ agree to exchange cost information in stage 1 and enter in stage 2. They then choose quantities in stage 3 under complete information according to (3.3). Firms $n_s^* + 1$ to $N$, meanwhile, stay out of the market in stage 2, in which case their decision on an information exchange in stage 1 is inconsequential.

- (4.1.2) If $n_s^* > n_{ns}^*$ and $E[\pi_s|n_s^*] \geq E[\pi_{ns}|n_{ns}^*]$, then firms exchange cost information.
In this instance, the gains in firms’ expected profits from the shift in information structure (from incomplete to complete information) dominates losses from additional entry. Hence, firms 1 through $n_s^*$ are better off under an agreement to exchange information and the equilibrium solution is identical to that in (4.1.1). The negative impact of entry decreases with the number of firms, while the gains from information sharing are greatest when the correlation in costs is low and products are close substitutes. Consequently, (4.1.2) is more likely hold when $(1 - \rho, b, n_s^*)$ are large.

This result is illustrated in graphs 3 and 4. These graphs have been constructed under the hypotheses that $n_s^* = n_{ns}^* + 1$ and $F_i < E[\pi_{ns} | n_{ns}^*] < F_{n_s^*} = E[\pi_s | n_s^*] \forall i = 1, ..., n_{ns}^*$. In other words, only the first $n_{ns}^*$ firms enter the market under incomplete information, and the firm $n_s^*$ does not make any profit under complete information. This last hypothesis implies that the graphs represent a worst case scenario with regard to the effects of information sharing on firms’ total profits. Graphs 3 and 4 indicate that the $n_s^*$ firms will want to share information when the pair $(\rho, b)$ is close to $(0, 1)$; i.e. when information is most valuable to firms, as explained earlier. This result would still hold true if firm $n_s^*$ had a smaller fixed cost that satisfied $E[\pi_{ns} | n_{ns}^*] < F_{n_s^*} < E[\pi_s | n_s^*]$. In addition, graphs 3 and 4 show that firms are more likely to find it profitable to share information when the number of active firms becomes larger.

- (4.1.3) If $n_s^* > n_{ns}^*$ and $E[\pi_s | n_s^*] < E[\pi_{ns} | n_{ns}^*]$, then firms do not exchange cost information.

By definition of $n_{ns}^*$ and $n_s^*$, $E[\pi_{ns} | n_{ns}^* + i] < F_{n_{ns}^*+i} \leq E[\pi_s | n_s^* + i] \forall i = 1, ..., (n_s^* - n_{ns}^*)$
and firms \(n_{ns}^* + 1\) to \(n_s^*\) only enter under an agreement to exchange cost information. However, firms 1 through \(n_{ns}^*\) are worse off, in expected terms, under such an agreement. Thus, in the equilibrium, (at least one of) these firms do not exchange cost information. Then, firms 1 through \(n_{ns}^*\) enter in stage 2 and choose quantities in stage 3 under incomplete information according to (3.1). Meanwhile, firms \((n_{ns}^* + 1)\) to \(N\) stay out of the market in stage 2.\(^{17}\)

In (4.1.3), additional entry lower firms’ expected profits, despite the shift in information structure (from incomplete to complete information). Additional entry hurts firms most when there are fewer firms, and when information sharing is less valuable to firms; that is, when products are highly differentiated and costs are highly correlated. As show in Graphs 3 and 4, firms therefore do not share information when \((b, 1 - \rho)\) is close to \((0,0)\) and there are few firms in the market. Note that when \(b = 1\) and \(\rho = 1\), we are in a standard Cournot model with complete information and perfect substitutes, and additional entry will also lower individual firms’ profits.

To construct examples satisfying (4.1.3), it suffices, given the definitions of \(n_{ns}^*\) and \(n_s^*\), to choose the fixed cost of firm \(n_{ns}^* + 1\) such that it satisfies \(E[\pi_{ns}|n_{ns}^* + 1] < F_{n_{ns}^* + 1} \leq E[\pi_s|n_{ns}^* + 1]\). As an illustration, consider the duopoly example in table 1 with these fixed costs: \(F_1 = 0.16\) and \(F_2 = 0.163\). We have that \(n_{ns}^* = 1\) since \(E[\pi_{ns}|n = 2] =\) \(0.162 < F_2\), and \(n_s^* = 2\) since \(F_2 < 0.166 = E[\pi_s|n = 2]\). As \(E[\pi_s|n = 2] = 0.166 <\)

\(^{17}\)Allowing for subsets of entrants to exchange information in stage 1 of the model does not change the essence of the results, but it may yield additional solutions in (4.1.3). Consider, for instance, a model with 3 firms such that \(n_s^* = 3\) and \(n_{ns}^* = 1\) and (4.1.2) holds. Then firm 1 (which has the lowest fixed cost) does not want to exchange information. Firms 2 and 3 may, however, agree to exchange with each other in which case, say, they enter. The equilibrium solution may then involve an exchange of information among all three firms.
0.25 = E [\pi_{ns}|n = 1], firm 1 has higher expected profits if it does not exchange information.\textsuperscript{18} Hence, in equilibrium, firms do not exchange information in stage 1 and only firm 1 enters in stage 2. In stage 3, it produces the monopoly output given its marginal cost.

In (4.1.3), an agreement not to exchange information must be binding.\textsuperscript{19} Indeed, if renegotiating ex-post entry decisions were possible, then the original agreement (in stage 1 of the game) would not be credible and would, thus, be meaningless. In that case, all \( n^*_s \) firms in (4.1.3) would invest in foreseeable expenses expecting (correctly) to exchange information after entry. This would, however, yield lower profits to the \( n^*_{ns} \) firms which originally decided not to share information. To ensure the highest possible profits for themselves, these \( n^*_{ns} \) firms must therefore credibly commit not to share information, even ex-post entry.\textsuperscript{20} Note that the existence of binding agreements is implicitly assumed in the literature on exchanges of information, since firms are typically not allowed to renegotiate their agreement after observing their own costs (see our discussion of the SOM in Section 2).

\textsuperscript{18}The monopoly equilibrium quantity is \( x = \frac{a-c}{2} \), and the expected profit is \( \frac{1}{4}E \left( (a-c)^2I_{[c \leq a]} \right) \), or equivalently, \( \frac{1}{4}E \left[ y^2 I_{[y \leq 0]} \right] \) where \( y \) is in our example a normal \( N(0,2) \). Given the properties of the normal distribution, we have \( E \left[ y^2 I_{[y \leq 0]} \right] = \frac{1}{2}E \left[ y^2 \right] = 1 \). Therefore, the expected profit of the monopoly in our example is equal to 0.25. The other numerical values are taken directly from Table 1.

\textsuperscript{19}Firms have no incentive to renegotiate an agreement to exchange information since, as we have seen in Section 3.1, firms always have an incentive to share information after entry.

\textsuperscript{20}The design of a mechanism leading to such a credible agreement is beyond the scope of the present paper.
4.2. Exchanges of Information: Consumer Welfare

Here, we show that consumers may benefit from exchanges of information across a wider range of market structures than implied by the SOM. We first note that when information sharing does not affect the number of entrants (i.e., \( n^*_s = n^*_ns \)), the consumer surplus analysis is equivalent to the one conducted in Section 3:

- (4.2.1) When \( n^*_s = n^*_ns \geq 2 \), consumers may benefit from information sharing for a larger set of parameters \((b, \rho) \in [0,1]^2\) than the SOM suggests.

In market structures with as few as two firms, consumers may benefit simply from the shift from incomplete to complete information, even when the number of entrants on the market remains constant (see graphs 1 and 2). Hence, information sharing may jointly improve the welfare of individual firms (see (4.1.1)) and consumers in duopoly and oligopoly markets.

We next consider the cases where the number of entrants is larger under information sharing.

- (4.2.2) If \( n^*_s > n^*_ns \), then consumers may benefit from information sharing.

In (4.2.2), fixed costs are such that information sharing not only changes the competitive information structure, but it also leads to additional entry. As demand is linear and marginal costs are constant in the SOM, entry of new firms benefits consumers under complete information. Hence, these additional gains are such that the range in parameter values \((b, \rho) \in [0,1]^2\) for which consumers now benefit expands beyond that in
(4.2.1) where information sharing did not change the number of firms.\textsuperscript{21} In particular, as illustrated in Graphs 3 and 4, the entry of new products most benefit consumers when there are few close substitutes to the new products; that is, when products are highly differentiated ($b$ close to 0) and there is a small number of active firms.

To illustrate (4.2.2), consider the duopoly example of Table 1 with $F_1 = 0.16$ and $F_2 = 0.163$, so that, as explained earlier, $n_s^* = 2$ and $n_{ns}^* = 1$. The expected consumer surplus is higher with information sharing (i.e. 0.240 when $n_s^* = 2$ vs. 0.125 when $n_{ns}^* = 1$).\textsuperscript{22} Note that consumers may even benefit when costs are uncorrelated, in contrast to findings in the SOM. For instance, assume that $\rho = 0$ and $F_2 = 0.21$, but all other parameters are unchanged, in our duopoly example of Table 1. Then, $n_s^* = 2$, $n_{ns}^* = 1$, and expected consumer surplus is again higher with information sharing (i.e. 0.245 when $n_s^* = 2$ vs. 0.125 when $n_{ns}^* = 1$).\textsuperscript{23}

- (4.2.3) If $n_s^* > n_{ns}^*$ and $E[\pi_s|n_s^*] < E[\pi_{ns}|n_{ns}^*]$, social welfare may be higher if firms were to exchange information.

In (4.2.3), firms do not exchange information since it reduces their expected profits and, thus, only firms 1 to $n_{ns}^*$ enter (see 4.1.3). If firms were to share information though, additional firms would enter and the increase in consumer surplus might more

\textsuperscript{21}The gains to consumers in these regions are larger than those that would result from entry alone (i.e. without a change in the information structure).

\textsuperscript{22}The value 0.240 is from Table 1 (i.e. C.I. with constraint). The consumer surplus in a monopoly market is equal to $x^2/2 = [(a - c)^2 I_{[c \leq a]}]/8$. Since, as explained earlier, $E[(a - c)^2 I_{[c \leq a]}] = 1$, the expected consumer surplus in our monopoly example equals 0.125.

\textsuperscript{23}When $\rho = 0$ in the example of Table 1, $E[\pi_s|n_s^* = 2] = 0.2257$ and $E[\pi_{ns}|n_{ns}^* = 2] = 0.2087$. Hence, when $F_1 = 0.16$ and $F_2 = 0.21$, then $n_{ns}^* = 1$ and $n_s^* = 2$. As explained in the prior footnote, expected consumer surplus when $n_{ns}^* = 1$ is 0.125. By simulation, we get that the expected consumer surplus when $n_s^* = 2$ now equals 0.245.
than compensate for firms’ losses, so that social welfare would be higher. For instance, in our duopoly example of table 1 with $F_1 = 0.16$ and $F_2 = 0.163$, social welfare would be higher if firms shared information (i.e. 0.249 vs. 0.215). As explained in (4.2.2), consumers mostly benefit when $(1 - b, \rho)$ is large, and graphs 3 and 4 indicate that social welfare would typically rise in these regions if firms were to exchange information.

This discussion therefore suggests that under certain circumstances, policy-makers may want to entice firms to exchange information. Such incentives could be provided with lump-sum taxes on the consumer’s income or on firms’ profits. More significantly, such incentives may actually involve no monetary transfers in equilibrium (as opposed to a simple subsidy of entry). Suppose, in our duopoly example, that policy-makers guarantee to provide a sum of 0.002 to firm 2 if it enters and does not exchange information. Then, if firms do not exchange information and firm 2 enters, it expects to earn net profits of $0.001 = 0.162 + 0.002 - 0.163$, in which case it enters. Since $n_{ns}^* = n_s^* = 2$, it is now optimal for both firms to share information (see (4.1.1)), in which case firm 2 earns net expected profits of 0.003 (i.e., $-0.163 + 0.166$). Thus, the equilibrium solution has firms sharing information in stage 1. Both consumers and society benefit and, since firms share information, policy-makers never actually subsidize firm 2. Note that both producer and consumer surpluses are higher under this policy than they would have been had policy-makers simply subsidized entry without allowing for information sharing.

\[ ^{24} \text{Under information sharing, expected net profits are equal to } 0.166 \times 2 - 0.163 - 0.16 = 0.009 \text{ and expected consumer surplus is } 0.24, \text{ so that expected social welfare equals } 0.249. \text{ Under no information sharing, expected social welfare is equal to } 0.25 - 0.16 + 0.125 = 0.215. \]
5. Concluding Remarks

We have shown that endogenizing entry affects predictions regarding both firms’ decisions to share cost information and the benefits of information sharing for consumers.\(^{25}\) In particular, our results indicate that firms may decide not to exchange information in a competitive Cournot model with cost uncertainty, even though such an exchange may improve consumer and social welfare. We have then shown that enticing firms to share information may not require any monetary transfer on the part of policy-makers. Such incentives could be provided through traditional channels (e.g. trade groups or research consortia), but also through support for B2B e-commerce platforms. Indeed, industry-specific B2Bs (e.g. the Covisint project for auto-makers, the Transora B2B for the consumer packaged good industry, and most subsidiaries of the VerticalNet B2B such as Aerospace Online, Bakery Online, Semiconductor Online, or Nurses.com) routinely provide members with industry newsletters, market research, discussion forums, and member associations.\(^{26}\) This class of B2Bs, known as “vertical marketplaces” to market insiders, appears therefore as a good medium to promote information exchanges within an industry.\(^{27}\)

\(^{25}\)The results in the present paper are not conditional upon a specific distribution for the marginal costs of production. Examples with different distributions are available on our web site: http://www.simon.rochester.edu/fac/richard/.

\(^{26}\)Transora is a B2B marketplace created in 1999 by 49 firms in the consumer packaged good industry (such as Coca-Cola, PepsiCo, Procter&Gamble, Kraft foods or Nabisco) to facilitate relations with their common suppliers.

\(^{27}\)These results hold without even considering the cost efficiencies that have been associated with B2B exchanges (i.e., inexpensive access to sales channels, reduction in administrative and supply-chain costs). In an interesting review of the Federal Trade Commission’s workshops, Robert Ploch and Scott Perlman, partners at the law firm Mayer, Brown & Platt in the area of antitrust, reach similar conclusions. Perlman was a member of the Commission’s panel and the report is available at www.mayerbrown.com.
From a consumer perspective, we find that information sharing may be beneficial in a wide range of market structures, including oligopolies with few firms (even duopolies). These results are interesting from a regulatory perspective, as they contrast with previous findings and current policy concerns regarding B2B marketplaces. In particular, our analysis suggests that, pending a competitive marketplace and a truthful exchange of information, B2B exchanges may encourage entry and, thereby, benefit consumers. Even if such exchanges do not expand the number of entrants, our findings indicate that they may make it more economical for firms to enter small markets that would otherwise be too expensive to supply. Indeed, as we have documented in Section 3, firms more frequently supply a market where marginal costs of production are high when they compete under information sharing.

To conclude, it is interesting to note that our model suggests that the B2Bs previously mentioned for promoting information sharing (i.e. Covisint, Transora, and VerticalNet subsidiaries) are also likely to benefit consumers. Indeed, in the automobile, consumer packaged good, or aerospace industries, costs are often intimately correlated, products are easily substitutable, and the creation of B2Bs is unlikely to generate the entry of new firms. As shown in Section 4, information sharing in industries with these characteristics may, under our model’s assumptions, improve consumer welfare.
6. References


7. Tables and Graphs

**TABLE 1: TWO FIRMS**

Example, based on the specifications outlined in Section 2, when \( a=10, \ b=0.5, \) and costs are jointly normally distributed with \( \mu = 10, \ \sigma^2 = 2, \) and \( \rho = 0.95 \)

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Profit</th>
<th>Entry</th>
<th>Consumer Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Var</td>
<td>Cov</td>
<td>Mean</td>
</tr>
<tr>
<td>With Constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.I</td>
<td>0.227</td>
<td>0.114</td>
<td>0.095</td>
<td>0.166</td>
</tr>
<tr>
<td>I.I</td>
<td>0.226</td>
<td>0.111</td>
<td>0.104</td>
<td>0.162</td>
</tr>
<tr>
<td>Without Constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.I</td>
<td>0.002</td>
<td>0.334</td>
<td>0.290</td>
<td>0.334</td>
</tr>
<tr>
<td>I.I</td>
<td>0.003</td>
<td>0.326</td>
<td>0.310</td>
<td>0.326</td>
</tr>
</tbody>
</table>

C.I and I.I respectively stand for Complete Information and Incomplete Information.

**TABLE 2: TEN FIRMS**

Example, based on the specifications outlined in Section 2, when \( a=10, \ b=0.5, \) and costs are jointly normally distributed with \( \mu = 10, \ \sigma^2 = 2, \) and \( \rho = 0.7 \)

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Profit</th>
<th>Entry</th>
<th>Consumer Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Var</td>
<td>Cov</td>
<td>Mean</td>
</tr>
<tr>
<td>With Constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.I</td>
<td>0.114</td>
<td>0.055</td>
<td>0.008</td>
<td>0.068</td>
</tr>
<tr>
<td>I.I</td>
<td>0.096</td>
<td>0.025</td>
<td>0.016</td>
<td>0.034</td>
</tr>
<tr>
<td>Without Constraint</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.I</td>
<td>7.1D-4</td>
<td>0.275</td>
<td>0.008</td>
<td>0.274</td>
</tr>
<tr>
<td>I.I</td>
<td>0.001</td>
<td>0.075</td>
<td>0.053</td>
<td>0.076</td>
</tr>
</tbody>
</table>

C.I and I.I respectively stand for Complete Information and Incomplete Information.
Under information sharing, we have:

**Region 1**: Consumer surplus is lower in the traditional and generalized SOMs.

**Region 2**: Consumer surplus is higher in the generalized SOM, lower in the traditional SOM.

**Region 3**: Consumer surplus is higher in the traditional and generalized SOMs.

**Region 1**: Firms exchange information, and social welfare is higher.

**Region 2**: Firms do not want to exchange information, but social welfare would be higher if they did.

**Region 3**: Firms do not want to exchange information, and social welfare would be lower if they did.

In Graph 3, **consumer surplus** is always higher under an exchange of information.

**Region 1 and 2**: Firms exchange information, and social welfare is higher.

**Region 3**: Firms do not want to exchange information, but social welfare would be lower if they did.

**Region 4**: Firms do not want to exchange information, but social welfare would be higher if they did.

In Graph 4, **consumer surplus** is only lower in **Region 1** under an exchange of information.

We provide an addendum with the details of the different equilibria in Sections 2 and 3 of the paper at the following web site: http://www.simon.rochester.edu/fac/richard/.

The numerical determination of the complete information problem in Section 3 is straightforward. We use standard numerical procedures to solve the system of non-linear equations (3.4) leading to the threshold costs $\bar{c}_i$. Then, we can determine the equilibrium quantities by substituting $\bar{c}_i$ in (3.3).

To determine equilibrium quantities in the incomplete information model, we need to solve the system of equations (3.2) and then (3.1). Note that (3.1) depends upon the function $E[x(c)|c_i]$. In this case, unlike a complete information setting, firms cannot predict the exact quantities that their rivals produce at the Nash solution. To determine their best strategies, firms can rely only upon their rivals’ conditional expected quantities, $E[x(c)|c_i]$. There is no analytically tractable way, however, to calculate the function $E[x(c)|c_i]$ at any point $c_i$.

We propose to replace this function by an approximation $f(c_i, \beta)$ parametrized by a vector $\beta$. Intuitively, $f(., \beta)$ is the fixed point solution of a problem matching a potential expected quantity to its empirical counterpart as calculated across Monte Carlo simulations. The algorithm proceeds as follows:

We simulate $S$ private types (using the Common Random Number technique) for the representative firm, $\{\tilde{c}_s\}_{s=1,...,S}$, from a normal distribution with mean $\mu$ and variance $\sigma^2$. For each value $\tilde{c}_s$, we then simulate $K$ conditional private types, $\{\tilde{c}_{s,k}\}_{k=1,...,K}$, from
a normal distribution with mean \( \mu (1 - \rho) + \rho \tilde{c}_s \) and variance \( \sigma^2 (1 - \rho)^2 \). Therefore, \( \tilde{c}_{s,k} \) may be interpreted as the private cost of an opponent of firm \( i \), conditional on the fact that firm \( i \) has a cost \( \tilde{c}_s \).

The determination of the approximation \( f (., \beta) \) proceeds in several steps. First, for a given approximation of the conditional expectation \( (f (., \beta)) \) we find \( \tau (\tilde{c}_{s,k}) \) and \( \bar{x} (\tilde{c}_{s,k} | \tilde{c}_s) \) for any \( \tilde{c}_{s,k} \) by solving numerically equations (3.2) and (3.1). Next, we calculate the conditional empirical mean \( \hat{E} [x (c) | \tilde{c}_s] \) of the quantity produced by an opponent of firm \( i \), when firm \( i \) has a cost \( \tilde{c}_s \). The object of the algorithm is then to find \( \beta \) such that \( f (\tilde{c}_s, \beta) \) is arbitrarily close to \( \hat{E} [x (c) | \tilde{c}_s] \) for any \( \tilde{c}_s \). In other words, the approximation \( f (., \beta) \) is the solution of

\[
\min_{\beta} \sum_{s=1}^{S} || f (\tilde{c}_s, \beta) - \hat{E} [x (c) | \tilde{c}_s] || \quad \text{where} \quad (8.1)
\]

\( f (\tilde{c}_s, \beta) \) is a piecewise linear function with parameters \( \beta \in \mathbb{R}^l \),

\[
\hat{E} [x (c) | \tilde{c}_s] = \frac{1}{K} \sum_{k=1}^{K} \bar{x} (\tilde{c}_{s,k} | \tilde{c}_s) \quad ,
\]

\[
\bar{x} (\tilde{c}_{s,k} | \tilde{c}_s) = \frac{1}{2} (a - b (N - 1) f (\tilde{c}_{s,k}, \beta) - \tilde{c}_{s,k}) I_{\{\tilde{c}_{s,k} < \tau (\tilde{c}_{s,k})\}} \quad ,
\]

\[
\tau (\tilde{c}_{s,k}) = a - b (N - 1) f (\tilde{c}_{s,k}, \beta) \quad .
\]

In practice, \( K = 10^4, S = 10^6, l = 5 \), and the precision for the minimization problem in (7.1) is of order \( 10^{-9} \). Once \( f (., \beta) \) has been determined, we can calculate the equilibrium quantities for a given cost vector \( c \).